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On Another Formulation of the de Broglie's Hidden Thermodynamics

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1. *Introduction*

Some recent investigations led Louis de Broglie (de Broglie, 1964b, 1968a, b) to the development of a new thermodynamics which he called the thermodynamics of the isolated particle. This theory is an important part of the causal formulation of wave mechanics (de Broglie, 1960, 1964a). Thus, it is the hypothesis of the existence of a chaotic deep medium, the so-called subquantum medium, which was introduced in wave mechanics (Bohm & Vigier, 1954) to explain the probability distribution, that led de Broglie to take into consideration the interaction of particles with a huge hidden thermostat.

As he did not consider the particle otherwise as a very simple physical system, where the meaning of its temperature and entropy could hardly be understood, de Broglie ascribed the temperature and entropy values to the particle in a somewhat indirect way, namely by means of the thermodynamic equilibrium of the particle with the hidden thermostat. Although this point of view has been well developed, we do not take it by any means as necessary.

As a matter of fact, and since this theory is only the thermodynamic aspect of the causal interpretation of wave mechanics, it must necessarily rest on the model of the particle describing it as a regular physical wave of very weak amplitude but with large values of the field in a small local region. Thus, underlying the new thermodynamics, there is the concept of the particle as a mathematical entity with a great number of degrees of freedom and we think it justified to assign directly to the particle not only a heat content but even a temperature and an entropy of its own. Obviously, the equilibrium conditions between the particle and the thermostat must be retained.

The aim of the present paper is to develop the thermodynamics of the isolated particle from this point of view. We shall see that it allows us to find once more the set of results already obtained. Nevertheless, this new approach provides some interesting extensions concerning the free energy which we shall now take as it is usually defined.

2. *Some Preliminary Results*

Before going further we must briefly summarise a certain number of results which become necessary later. The reader is referred to the bibliography for further detail.

According to the causal formulation of wave mechanics, the corpuscle, i.e. the small region where the field takes very great values and where nearly all the energy is concentrated, is moving in the physical wave v according to a certain 'guidance law' which can be obtained from the wave equation. Using the Klein-Gordon equation, that guidance law will take the form

$$\mathbf{v} = -c^2 \frac{\nabla\varphi + (e/c)\,\mathbf{A}}{(\partial\varphi/\partial t) - eV} \tag{2.1}$$

where $\mathbf{v}$ is the velocity of the corpuscle and φ is the phase of the wave v (or that of the statistical wave Ψ, which is proportional to v) written in the form $v = a\exp(i\varphi/\hbar)$. It can be shown that the later expression for the velocity may also be obtained by means of Jacobi's classical theorem applied to the generalized Hamilton-Jacobi equation

$$\frac{1}{c^2}\left(\frac{\partial\varphi}{\partial t} - eV\right)^2 - \sum_{xyz}\left(\frac{\partial\varphi}{\partial x} + \frac{e}{c}A_x\right)^2 = m_0{}^2c^2 + \hbar^2\frac{\Box a}{a} \tag{2.2}$$

formally deduced by a simple decomposition of the wave equation. Equation (2.2) generalises the well-known classical equation by introducing the quantum potential

$$Q = -\frac{\hbar^2}{m_0}\frac{\Box a}{a} \tag{2.3}$$

which one may interpret as expressing the action of wave v upon the motion of the corpuscle. It is the purely quantum force deriving from this potential that is, for instance, responsible for diffraction phenomena occurring at the neighbourhood of the border of a screen.

It can be seen that the motion of the corpuscle is also governed by the Lagrangian

$$\mathscr{L} = -M_0c^2\sqrt{(1-\beta^2)} - eV + \frac{e}{c}\mathbf{v}.\mathbf{A} \tag{2.4}$$

where one considers M_0 as the proper mass *varying* according to

$$M_0 = \left(m_0{}^2 + \frac{\hbar^2}{c^2}\frac{\Box a}{a}\right)^{1/2} \tag{2.5}$$

where m_0 is the usual proper mass. Taking the non-relativistic approximation, one may write

$$M_0c^2 - m_0c^2 = Q \tag{2.6}$$

and Q becomes

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 a}{a} \tag{2.7}$$

as can be easily verified by decomposition of the Schrödinger equation.

We have pointed out at the outset, that overlapping with this deterministic scheme there is within the present theory a hypothesis of random motion, i.e. the postulate of the interaction of the subquantum medium and the particle. It follows that we can not know the position of the corpuscle at each instant, and consequently we do not know the value which $M_0(\mathbf{r}, t)$ takes at the same instant. Besides, that value generally undergoes a very quick change in time, so that we can only calculate its mean value taken over the corresponding probability distributions.

We shall not summarise here the work (Helmholtz, 1884; Boltzmann, 1904) relating to the analogies between mechanical and thermodynamical quantities without interference of statistical concepts, although it turns out to be most revealing within the framework of the ideas given above. A brief résumé will be given later about the canonical scheme of Helmholtz. For the moment we just want to point out that Ludwig Boltzmann in his theory of periodic systems (which is the source of Ehrenfest's theory of adiabatic invariants) proved the formula

$$\delta \mathscr{Q} = \frac{1}{\tau} \delta \oint \sum_k p_k \, dq_k \tag{2.8}$$

where $\delta \mathscr{Q}$ is the amount of heat supplied to the periodic system during the period τ and $A = \oint \sum p_k dq_k$ is Maupertuis' action taken over that same period. The reader may find in a paper of Fer (1962) a detailed study of this formula.

We still want to refer to some points of relativistic thermodynamics, namely to emphasise that in the domain studied here the exactness of the formulae for relativistic transformation of heat and temperature

$$\mathscr{Q} = \mathscr{Q}_0 \sqrt{(1-\beta^2)}; \qquad T = T_0 \sqrt{(1-\beta^2)} \tag{2.9}$$

will not be questioned (de Broglie, 1968a, 1968b; Brotas, 1969). A word must also be said about the equation

$$\delta \mathscr{Q} = -\delta_{M_0} \mathscr{L} \tag{2.10}$$

for it turns out to be necessary later. It is an equation in relativistic thermodynamics, closely connected to the Einstein relation between energy and mass, and it expresses the variation of the Lagrangian $\mathscr{L}$ of a given system by the heat supply $\delta \mathscr{Q}$ responsible for the corresponding variation of the proper mass M_0; $\delta_{M_0} \mathscr{L}$ is the variation of $\mathscr{L}$ when all the variables but M_0 are kept constant.

3. *Fundamental Formulae*

The starting point of wave mechanics was the concept of the corpuscle as a system endowed with an internal periodic motion whose frequency (cyclic frequency) changes in a relativistic transformation according to the law

$$\nu_c = \nu_0 \sqrt{(1-\beta^2)} \tag{3.1}$$

whereas the transformation law of a wave of frequency ν is

$$\nu = \frac{\nu_0}{\sqrt{(1-\beta^2)}} \tag{3.2}$$

Besides, guidance laws [as that written in (2.1)] arise from the requirement of an accordance of phases between the corpuscle and its wave in all the Galilean frames of reference.

It appears to be most tempting to apply Boltzmann's formula (2.7) to the internal cyclic motion of the corpuscle, and doing so we shall be led to establish a proportionality between Maupertuis' action A and the amount of heat absorbed by the corpuscle, the constant of proportionality being the proper frequency ν_0. Since $\delta\mathcal{Q} = T\delta S$, write

$$\nu_c = CT, \qquad S = CA_0 \tag{3.3}$$

where, according to dimensional reasons, the universal constant C is proportional to k/h. The equation

$$h\nu_c = kT \tag{3.4}$$

setting up a connection between the cyclic frequency and the temperature of the corpuscle is the first fundamental formula of the new thermodynamics.

Yet the second formula in (3.3) is not a relativistic covariant one and that is why de Broglie altered it by changing Maupertuis' action into the Hamiltonian action

$$A = \int_0^T \mathcal{L}\, dt \tag{3.5}$$

We notice the consistency of this reasoning with the relativistic formula (2.10) and from (1.5) it follows thence

$$\delta_{M_0} S = \frac{\delta\mathcal{Q}}{T} = -\frac{h\nu_c}{k}\delta_{M_0}\mathcal{L} = k\frac{\delta M_0}{m_0} \tag{3.6}$$

which leads us to write the second fundamental formula

$$S = S_0 + k\frac{M_0}{m_0} \tag{3.7}$$

where S is the entropy of the corpuscle, S_0 the part of this entropy that does not depend upon the variations of the proper mass, and M_0 is given, for instance, by (2.6). The physical meaning of this formula will be discussed later.

4. *Discussion of the Fundamental Formulae*

The temperature T defined by (3.4) is ascribed by de Broglie to the hidden thermostat with which the particle lies in thermodynamic equilibrium, whereas we intend to relate this temperature directly to the corpuscle itself, which has cyclic frequency ν_c. In these conditions, and by (2.9) and (3.1), the relativistic covariance of equation (3.4) raises no more doubts, but the questions relating to the thermodynamic equilibrium between the thermostat and the particle, namely the equality of their temperatures, call for a more detailed examination.

First of all, the possibility of defining a velocity of the corpuscle in respect to the thermostat is certainly meaningless, since otherwise the thermostat would constitute a privileged frame of reference, in disagreement with the principle of relativity. The thermostat seems, then, to have physical properties similar to the ether of Dirac (1951) or the ether of Terletski (1960a, b) rather than Kelvin's ether. It follows that any two Galilean observers must see the thermostat in exactly the same way, and that means that everything turns out to be as if all the corpuscles were stationary in respect to the thermostat.

The same conclusion may be obtained from the proposal of Brotas, according to whom thermodynamic equilibrium of two bodies in relativity must be defined in terms of the equality of their proper temperatures (Brotas, 1969). If we adopt this definition here it follows from the relativistic structure of the thermostat that its temperature must be the same in all Galilean frames (this is its proper temperature) and hence the thermodynamic equilibrium with the corpuscle is assured whatever the corpuscle's state of motion may be.

Another interesting problem is the following: the proper temperatures of two non-identical corpuscles are necessarily different, for they correspond to different proper masses. The equilibrium with the thermostat implies, therefore, that it must behave with respect to these two corpuscles as if it had two different temperatures simultaneously, which is not very satisfactory. De Broglie suggested that this might be due to some 'resonances', where each corpuscle would only interact with some component parts of the thermostat, perhaps only with other hidden corpuscles of the same nature.

We wish to point out that this quite singular property seems to be closely connected with the very nature of relativistic wave mechanics. This may be clarified by noting, for instance, that Dirac's theory involves the simultaneous existence of an ocean of positrons, another of antinucleons, etc., the whole ensemble being quite similar to the hidden thermostat. Developing a suggestion of Lochak (1968), we point out that the new thermodynamics can be reformulated in such a way that one may ascribe to the thermostat a unique temperature, which would then be the proper temperature of any given particle. That could be done by writing the constant C appearing in formulae (3.3) in the form

$$C=\frac{m_0 k}{\mu_0 h} \tag{4.1}$$

where m_0 is the proper mass of the particle in question and μ_0 a standard mass fixing the temperature of the thermostat. In place of (3.4) we have

$$h\nu_c=\frac{m_0}{\mu_0}kT \tag{4.2}$$

whereas (3.7) now becomes

$$S = S_0 + k\frac{M_0}{\mu_0} \tag{4.3}$$

At the moment, it seems too early to choose from these possibilities and, after all, this choice is needless as far as the following results are concerned.

Let us finally mention (de Broglie, 1968b) the striking agreement between the classical formula of Planck-Laue written in the form

$$U = \frac{M_0 c^2}{\sqrt{(1-\beta^2)}} = M_0 c^2 \sqrt{(1-\beta^2)} + \mathbf{v}.\mathbf{p} \tag{4.4}$$

and the guidance theory which allows us to write

$$\frac{\partial \varphi}{\partial t} = \frac{M_0 c^2}{\sqrt{(1-\beta^2)}} \qquad \nabla \varphi = -\frac{M_0 \mathbf{v}}{\sqrt{(1-\beta^2)}} \tag{4.5}$$

We thus have

$$M_0 c^2 \sqrt{(1-\beta^2)} = \frac{\partial \varphi}{\partial t} + (\mathbf{v}.\nabla\varphi) = \dot{\varphi} \tag{4.6}$$

and, if we ascribe to the corpuscle the main properties of a watch of internal proper frequency M_0c^2/h, the corresponding phase will be

$$\varphi_1 = h\nu_0 \sqrt{(1-\beta^2)}\, t = M_0 c^2 \sqrt{(1-\beta^2)}\, t \tag{4.7}$$

which means

$$d(\varphi - \varphi_1) = 0 \tag{4.8}$$

5. *The Helmholtz' Canonical Scheme*

The canonical scheme of thermodynamics proposed by Helmholtz is essentially based on the introduction of a cyclic variable ε defined in such a way that the temperature T is its time derivative:

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = T \tag{5.1}$$

Let $\mathscr{E}$ be the generalised force corresponding to ε. We then have for the variation of the energy U

$$dU = \mathscr{E}\, d\varepsilon - p\, dV = \mathscr{E}\dot{\varepsilon}\, dt - p\, dV \tag{5.2}$$

Comparing this expression with $dS = (dU + p\,dV)/T$ we have, by (5.1),

$$\mathscr{E} = \dot{S} \tag{5.3}$$

If we now consider a very slow evolution over time ($\dot{V} \cong 0$), the variational equations of motion with the Lagrangian $\mathscr{L}(V, \dot{V}, \dot{\varepsilon})$ shall be

$$\frac{\partial \mathscr{L}}{\partial V} = p \qquad \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\varepsilon}}\right) = \mathscr{E} \tag{5.4}$$

and that implies

$$p_\varepsilon = S \tag{5.5}$$

The expression for the energy will take the form

$$U = \dot{\varepsilon}\frac{\partial \mathscr{L}}{\partial \dot{\varepsilon}} + V\frac{\partial \mathscr{L}}{\partial \dot{V}} - \mathscr{L} = TS - \mathscr{L} \tag{5.6}$$

which means that the Lagrangian is equal to the free energy with a sign change. This last formula is well known, owing to its role in early investigations undertaken by Planck on black body radiation.

Yet the value of this canonical scheme may be questioned as long as it rests upon the introduction of a purely formal variable. Now, the thermodynamics of the isolated particle allows us to find again the relations obtained by Helmholtz, and even to attach to that variable a very clear physical meaning.

In fact, and according to the causal formulation of wave mechanics, we may write

$$\varphi = h\int_0^t \nu_c \, dt \tag{5.7}$$

and if we take (3.4) into consideration we shall have

$$T = \frac{1}{k}\dot{\varphi} \tag{5.8}$$

which is a relation corresponding to the fundamental formula (5.1). It is the phase of the wave v that here plays the role of the variable ε. If the particle gets some energy from the increase of its proper mass, this amount of energy may be expressed by the relation

$$\delta_{M_0} U = \mathscr{E}\, d\varphi + dA \tag{5.9}$$

$\mathscr{E}$ being the generalised force corresponding to the variable φ. By comparison with the first law of thermodynamics, we obtain $\mathscr{E} = \dot{S}$ that is (5.3).

Similarly, Helmholtz formula (5.4) is simply the consequence of the fact that φ is actually a cyclic variable. The equation (5.6) may not hold here, since it is not satisfactory relativistically.

6. *Hamilton's Principle and the Second Law of Thermodynamics*

Hamilton's principle of least action is generally described by comparing a real motion with very close neighbouring fictitious motions, so that the extreme points A and B, as well as the corresponding instants t_0 and t_1, remain unchanged. The real motion is then defined by the conditions

$$\int_{t_0}^{t_1} [\delta \mathscr{L}]_{M_0} dt = 0 \qquad \int_{t_0}^{t_1} [\delta^2 \mathscr{L}]_{M_0} dt > 0 \tag{6.1}$$

where $[\delta \mathscr{L}]_{M_0}$ denotes the variation of $\mathscr{L}$ when M_0 is kept constant.

Now, the variation of the proper mass brought about by fluctuations is a main property of the new thermodynamics, and this leads us to consider those varied paths no longer as virtual paths but rather as paths which may actually come about in view of adequate variations of proper mass. These must then obey the equation

$$\int_{t_0}^{t_1} \delta(\mathscr{L} + \delta\mathscr{L})\,dt = \int_{t_0}^{t_1} (\delta\mathscr{L} + \delta^2\mathscr{L})\,dt = 0 \tag{6.2}$$

But under these assumptions proper mass is no longer a constant in time, and then, putting $\delta_{M_0}\mathscr{L}$ for the part of $\delta\mathscr{L}$ depending upon the variation of proper mass, we must write

$$\delta\mathscr{L} = [\delta\mathscr{L}]_{M_0} + \delta_{M_0}\mathscr{L} \qquad \delta^2\mathscr{L} = [\delta^2\mathscr{L}]_{M_0} + \delta^2_{M_0}\mathscr{L} \tag{6.3}$$

We may later verify in equation (6.5) that the term $\delta^2_{M_0}\mathscr{L}$ is of the third order and may be neglected; the equation (6.2) becomes

$$\int_{t_0}^{t_1} (\delta_{M_0}\mathscr{L} + [\delta\mathscr{L}]_{M_0} + [\delta^2\mathscr{L}]_{M_0})\,dt = 0 \tag{6.4}$$

The second term being null, according to the first equation (6.1), we obtain

$$-\int_{t_0}^{t_1} \delta_{M_0}\mathscr{L}\,dt \equiv -(t_1 - t_0)\,\overline{\delta_{M_0}\mathscr{L}} = \int_{t_0}^{t_1} [\delta^2\mathscr{L}]_{M_0}\,dt > 0 \tag{6.5}$$

the inequality arising from the second equation (6.1). One then verifies that the mean value of $\delta_{M_0}\mathscr{L}$ taken over the time interval (t_0, t_1) is negative over the fluctuated paths while it is null over the real one.

According to (2.10) we know $-\delta_{M_0}\mathscr{L}$ to be the amount of heat supplied to the particle, and the preceding conclusion may be interpreted by asserting that the mean proper mass of the particle has a less value over the real path than over the fluctuated ones. The entropy of the thermostat will then have a maximum value over that path, and, in view of Boltzmann's formula, it must be then more probable than fluctuated paths. An interesting connection is thus established by de Broglie between the principle of Carnot and that of least action.

If we now reason directly about the particle itself, the equation (3.7) establishes that the entropy of the particle has a minimum value over the real path. But we must now take into consideration (Gibbs, 1902; de Broglie, 1964b) Gibb's formula $P = Ce^{-S}$, where S is here the entropy of the particle, instead of Boltzmann's, and this just shows that the preceding conclusion holds true. Then, when we ascribe thermodynamic quantities directly to the particle, we always have a maximum value of the probability over the real path.

7. *The Definition of the Free Energy*

We shall finally consider the important problem of the definition of the free energy whose properties must rule the thermodynamical behaviour of the particles (Lochak, 1962).

Since he did not directly attach an entropy and a temperature to the particle itself, de Broglie could not keep the usual expression of the free energy and that is why he was led to write

$$F = E_c + V - TS \tag{7.1}$$

where E_c is the kinetic energy and V the classical potential energy. We here no longer have the same conditions and we shall instead adopt the classical definition

$$F = U - TS \tag{7.2}$$

U, T and S being, respectively, the internal energy, the temperature and the entropy of the particle. Now we must write

$$U = m_0 c^2 + E_c + V + Q \tag{7.3}$$

and for any reversible transformation (being necessarily an isothermal one) it becomes

$$\delta F = \delta(m_0 c^2 + E_c + V + Q) - T\delta S = \delta(E_c + V + Q) - \delta\mathcal{Q} \tag{7.4}$$

where $-\delta\mathcal{Q}$ is the amount of heat supplied by the particle.

According to a demonstration of de Broglie which remains true here, we know the amount of heat $\delta\mathcal{Q}$ received by the particle to be equal to the variation δQ of the proper mass arising from that supply of heat

$$\delta\mathcal{Q} = \delta Q \tag{7.5}$$

and we then have

$$\delta F = \delta(E_c + V) \tag{7.6}$$

One may easily see that $\delta(E_c + V)$ is just the mechanical work performed by the particle, for the expression of the first principle $\delta U = \delta\mathcal{Q} - \delta A$, (7.3) and (7.5) give $-\delta A = \delta(E_c + V)$. We thus find the result

$$\delta F = -\delta A \tag{7.7}$$

and that means that the work δA performed by the particle corresponds to a loss of exactly the same amount of its free energy.

These considerations remain essentially unchanged for the irreversible transformations; If we write that $\delta\mathcal{Q} < T\delta S$ we shall find that

$$\delta F < -\delta A \tag{7.8}$$

We know from the second law of thermodynamics that an isothermal system performs spontaneously its evolution in the sense of a loss of its free energy. It follows thence that in quantum systems the jump of an electron from an energy level E_m to another level $E_k \neq E_m$ may take place spontaneously if $E_m > E_k$, since it then involves a decrease of the free energy of the system. The stationary state with greatest stability is obviously the one with lowest value of its free energy, that is, the fundamental state.

We may easily find this general result under an explicit form in the particular cases formely studied by de Broglie. Thus, for the linear oscillator and for the s-states of the hydrogen atom, $E_c = 0$ and

$$\delta A = -\delta(E_c + V) = -\delta V \tag{7.9}$$

But for these systems $V + Q = \text{const.}$, $\delta V = -\delta Q$ and then $\delta A = \delta Q$. Taking (7.5) into consideration, we may write

$$\delta A = T\delta S \tag{7.10}$$

and that means that when work is performed by the particle there is a decrease of the potential energy, an increase of the energy corresponding to the quantum potential and an increase of the entropy. The free energy always decreases.

It may be interesting to notice that when the kinetic energy is zero, δS and δF are then proportional to one another, the ratio of the two quantities being $-1/T$, any increase of S corresponding always to a decrease of F.

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